

Exploring the Applications of the Lorentz Force in Electromagnetic Propulsion

Sripaadh Jayashree Kuppasamy
Wakeland High School, 11th
Grade
sripaadhj@gmail.com

Abstract – *There exist two applications of the Lorentz force in electromagnetic propulsion: the Lorentz Rail Accelerator and the magnetoplasma dynamic thruster. For rail accelerators inductance gradient reigns supreme. For both, the apparatus' geometry is significantly important. This paper aims to unify two theoretical perspectives of the application of the Lorentz force.*

Index Terms - Lorentz force, Lorentz rail accelerator (LRA), magnetoplasma dynamic thruster (MPDT)

I. INTRODUCTION

In recent years, our dependence on the usage of fossil fuels has increased exponentially. Especially in the fields of rocketry and propulsion, the usage of traditional methods of chemical propulsion reign supreme. Regardless of the ubiquity of chemical combustion, electromagnetic propulsion offers its own benefits. The maximum theoretical specific impulse for a chemical propulsion rocket is about 500 s, as compared to the 5000 s (and theoretically 15,000 s) that electromagnetic propulsion can provide [1].

Electromagnetic launch systems, such as Lorentz Rail Accelerators (LRAs) and magnetoplasma dynamic thrusters (MPDTs) have emerged as promising new developments in the scope of projectile acceleration and propulsion. The underlying similarity between these two forms of electromagnetic propulsion is the Lorentz force. In Lorentz Rail Accelerators, an armature made of a conductive material is accelerated along a pair of current-conducting rails. In magnetoplasma dynamic thrusters,

ionized gas is accelerated through a discharge chamber, generating thrust.

Despite this similarity, these two forms of propulsion diverge in their application of the Lorentz force and operational environments. Lorentz Rail Accelerators cannot be used to propel masses like rockets in the traditional sense. For example, a theorized application of the LRA is on the Lunar surface [3]. On the other hand, magnetoplasma dynamic thrusters, which have extremely high efficiency, are useful for propulsion of rockets and such in space [1].

This paper aims to explore both applications of the Lorentz force under a unified framework, comparing physical principles and design.

II. THEORY

I. Lorentz Force

The main governing principle behind both magnetoplasma dynamic thrusters and Lorentz Rail Accelerators is the Lorentz force. This principle explains the force that is created due to the interaction between a moving charge (or current) and a magnetic field. This force is experienced by a charged particle or a current carrying conductor in a magnetic field.

II. Magnetic Field Generation

The magnetic field, $\mathbf{B}$, in these systems can be produced in two ways: by the current flowing through a plasma or a rail, or implementation of strong external magnetic fields. One method of magnetic field generation

may be more suited for one type of electromagnetic propulsion over the other.

The LRA generates magnetic fields produced by electrical currents according to the Biot-Savart law:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2} \quad (1)$$

where I is the current, μ_0 is the permeability of free space, $d\vec{l}$ is an increment of an object, $\hat{r}$ is the unit vector, and r is the displacement from the object. In systems with multiple current carrying conductors, the total magnetic field strength vector is approximately equal to the vector sum of each magnetic field from each conductor. This principle allows for a simplified estimation of the magnetic field strength in more complex geometries, such as the parallel rails in an LRA.

III. Lorentz Force in Plasma

Applying the same principle of magnetic force density, a specific Lorentz force equation can be derived. For a distributed current density $\mathbf{J}$ in a plasma, as seen in a magnetoplasmadynamic thruster, the Lorentz force is given by a volume integral:

$$\mathbf{F} = \int_V \mathbf{J} \times \mathbf{B} dV \quad (2)$$

where V is the volume of plasma. This integral allows us to account for the distributed nature of currents in ionized media, such as plasma, which is crucial for understanding MPDTs.

III. LORENTZ RAIL ACCELERATORS

I. Overview of Rail-Based Electromagnetic Acceleration

Lorentz Rail Accelerators operate by driving a large current through parallel conductive rails and an armature made of a conductive material. By Ampere's law, the flow of current through a conductive material creates a magnetic field perpendicular to the current. The generation of this magnetic field results in a Lorentz force on the armature, propelling it down the rail. Unlike Gauss guns, Lorentz Rail Accelerators often rely on self-generation, where the magnetic field that results in the Lorentz force is only one that is supplied by the current rather than outside sources, such as applied magnetic systems [4].

II. Inductance Gradients and the Lorentz Force

A key parameter in electromagnetic launchers is the inductance gradient, which is the rate of change of inductance with respect to distance along a system. In the

case of a Lorentz Rail Accelerator, the inductance gradient will be the change in inductance over the distance of a rail. The inductance gradient is crucial because it describes the relationship between the changing inductance and the armature's position along the rail, which is extremely important for determining the overall Lorentz force acting on the armature and the efficiency of the launcher. Specifically, the inductance gradient is directly related to the magnitude of the thrust due to the Lorentz force, which is related to the final velocity of a projectile. The inductance gradient of a rail can be affected by various factors, such as eddy current loss, armature position along a rail, and current diffusion [5].

Determining the inductance gradient is challenging and is often done using two methods: an analytical calculation done based on Biot-Savart's law and a solution method based on a finite element simulation. Since it is easier to make multiple calculations through the analytical approach, Ren et al. used a mainly analytical method to determine inductance gradient.

Using Biot-Savart's law to model magnetic fields, Ren et al. [5] deduced an analytical equation for the inductance gradient:

$$L' = \frac{2F}{I^2} \quad (3)$$

where L' is the inductance gradient (dL/dx), F is the force, and I is the current. This equation suggests that the Lorentz force is quadratically related with current and linearly related to L' . Realistically, the force will not be constant. Rather, it is likely that it is a function of x , or the armature position along a rail, and time. This slightly modifies the equation presented above when solving for the force:

$$F(x, t) = \frac{1}{2} I^2 L'(x, t) \quad (4)$$

Determining the inductance gradient requires consideration of various outside factors as well, such as skin effect, magnetic diffusion, and eddy current loss.

When a current is injected into a rail, its diffusion is not uniform. The current density, and therefore current, obeys the magnetic diffusion equation. Since, in a Lorentz Rail Accelerator, a DC current is switched on for a very short amount of time, the rate of change of the current ($\partial J / \partial t$) is very high. This results in a magnetic field that has not yet penetrated the material, and the magnetic field strength and the rate of change of current of the core of the rail is almost 0, resulting in extremely uneven distribution of current density near the surface of the rail.

The concentrated nature of the current at $t \approx 0$ results in a large inductance gradient, which will begin to reduce as the current density diffuses as time increases.

Ren et al. [5] confirms this phenomenon with a FEM simulation, which showed concentration of current density near the surface of the rails rather than the middle. Additionally, as the armature made its way down the rail, the inductance gradient decreased [5].

The rapid rise in current in the rails creates strong magnetic fields, which then induce eddy currents in the rails. These eddy currents can also contribute to the skin effect. As a large current is injected into the rail, a magnetic field is created, inducing a magnetic flux. A change in magnetic flux results in the creation of eddy currents. These currents circulate within the rail opposing the main current and push the current towards the rail surface.

III. Thermal Effects and Control Methods

Due to the transient skin effect, the current density of $\mathbf{J}$ is initially concentrated at the surface of the rail. Therefore, the rails are going to experience high levels of Joule heating.

For short, high-current pulses used in Lorentz Rail Accelerators, this leads to extreme temperature rises, which could lead to deformation or melting. These deformations could result in a decrease in the Lorentz Rail Accelerator's effectiveness and/or safety.

A remedy for this problem would be a cooling system. Currently, there are various methods for cooling: liquid cooling, air cooling, jet impingement cooling, micro-channel cooling, and spray cooling. Among these methods, Ma & Ma [6] found that spray cooling was the most effective due to its heat removal capacity. The angle at which the coolant is sprayed also has an impact on the degree to which the rail is cooled. When sprayed vertically, the heat flux value reached its highest value 12 seconds after spraying began. In the process of increasing spray angle, the critical heat flux decreased dramatically, with it having its lowest value at 5 degrees from the vertical. At this position, the heat flux was only 89% of the benchmark value [6]. However, the uniformity of surface temperature is highest when the coolant is sprayed from the vertical position. This is because when sprayed from an angle, the contact area of the droplets of coolant is larger than the contact area when sprayed vertically [6].

IV. Rail-Armature Geometry

The behavior of a Lorentz Rail Accelerator is heavily dependent on the geometry of the armature. Saravana Kumar and Murugan [8] found that the cross-sectional shape of the rails affected inductance gradient as well. Specifically, out of Rectangular, Rectangular-Circular Convex, Rectangular Concave, Rectangular-Semi Concave, and Circular Convex, the rails with a circular convex had comparatively higher inductance gradient, current density, and resultant force as shown in [8].

However, the application of this rail geometry is vague, as it is not clear how the rails would stay up straight without rolling due to their circular shape. Therefore, while the Rectangular-Circular Convex rails are not as effective as fully circular convex rails, they seem more practical, as they have a more stable base.

Therefore, in my model for a potential type of a Lorentz Rail Accelerator, I used a rectangular circular convex rail geometry with some extra modifications.

V. Model Lorentz Rail Construction

As it has been established that the most influential factor in a rail accelerator is the inductance gradient, I focused my attention on creating a rail model that maximizes the inductance gradient. With respect to the armature, the inductance gradient is dependent on the relative position between the contact leading edge and the root trailing edge [7]. Essentially, the thickness of the end of a C-type armature is influential in determining the lowest inductance path, or the path with the least magnetic energy. Because the thickness of the end of the armature determines the strength of the magnetic field, the relative position between the contact leading edge and the root trailing edge is important in determining inductance. Once an optimal thickness is determined, the effect of rail spacing will be observed through iterative simulations.

IV. SELF-FIELD MAGNETOPLASMADYNAMIC THRUSTERS

I. Overview of Magnetoplasma-dynamic Thrusters

The second application of the Lorentz force is found in magnetoplasma-dynamic thrusters [9]. These are a class of high-power electric propulsion devices that accelerate plasma via electromagnetic forces. While conventional chemical propulsion methods use chemical energy, MPD thrusters use electric energy to propel plasma, resulting in high exhaust velocities and could potentially lead to

longer-duration interplanetary missions. As stated before, the core principle of the MPD thruster is the Lorentz force. However, in MPD thrusters, the integral form of the Lorentz force is used, as it accounts for the distributed nature of currents in ionized media.

An MPD thruster typically has a coaxial anode-cathode arrangement in which a high-current, low-voltage discharge is initiated through a propellant. This discharge is often in the form of inert gases such as Argon or Krypton due to their lack of reactivity relative to other elements. These inert gases are transformed into ions, which are accelerated along the thruster axis, generating thrust.

MPD thrusters are an attractive concept because they promise high thrust and high specific impulse.

II. Self-Field Magnetoplasma dynamic Thruster Geometry and Configuration

The key distinction of the self-field MPD thruster is that there is no super imposed magnetic field. The magnetic field that allows for the Lorentz force is one that is created by the discharge current, enabling electromagnetic propulsion. Such a thruster consists of an annular anode on the outside, a cylindrical cathode in the center, and an insulated backplate. The discharge current flows radially from the cathode to the outer anode to create a complete circuit [10].

The self-created magnetic field from this current is mainly azimuthal ($\mathbf{B}_\theta$), resulting in a Lorentz force acting on quasi-neutral plasma. In this context, quasi-neutral plasma assumes that there is an equal number density of positively charged and negatively charged particles, resulting in an overall charge density equal to zero. Though there may be microscopic imbalances, they are quickly neutralized by the collective behavior of the particles, hence *quasi-neutral*.

Due to the Lorentz force, this plasma is thrust outwards, resulting in forward motion for an apparatus. However, since the self-field MPD thruster produces relatively small thrust, it is more suitable for low thrust applications such as satellites.

III. Lorentz Force Acceleration Mechanism

The principal acceleration mechanism in self-field thrusters is the interaction between the radial discharge current $\mathbf{j}_r$, and the self-created magnetic field $\mathbf{B}_\theta$. This interaction produces a Lorentz force density $f_z \approx \mathbf{j}_r \mathbf{B}_\theta$ directed downstream of the current.

The specific composition of the accelerated plasma is electrons and positively charged ions. Therefore, the momentum equations for electrons and ions are also able to be derived [1]:

$$\text{Ions: } m_i n \frac{D\mathbf{u}_i}{Dt} = ne(\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \nabla p_i + \mathbf{P}_{ei} \quad (5)$$

$$\text{Electrons: } -ne(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \nabla p_e + \mathbf{P}_{ei} \quad (6)$$

where m_i is ion mass, n is the plasma density, e is the charge of an electron, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{u}_i$ and $\mathbf{u}_e$ are the ion and electron velocities, p_i and p_e are the ion and electron pressures, and $\mathbf{P}$ is the momentum exchange term between the ions and electrons [1].

While these equations are technically momentum equations, it is possible to reduce them to net force equations using the differential form of Newton's Second Law ($\mathbf{F} = \frac{d\mathbf{p}}{dt}$). Using this law, the sum of all external forces can be equated to the rate of change of momentum. Therefore, it can be said that the change in momentum per unit volume is equal to the sum of all the external forces acting on the plasma. Through this connection, we are able to understand that the Lorentz force is not the only force acting on the plasma in an MPD thruster.

IV. MPD Thrust

To date, the self-field magnetoplasma dynamic thruster is known for a specific impulse in the range of 1500-1800s, with efficiencies exceeding 40% [11]. However, as per Choueiri [11], efficiencies greater than 30% (classified as high efficiency) are typically only reached at power levels greater than 100kW. Therefore, the steady state MPD Thruster has become an attractive form of propulsion, due to its capability of processing high levels of power, while providing high thrust densities.

An analytical model for the thrust of a self-field magnetoplasma dynamic thruster was developed by Maecker, under the assumption of an ideal version of the MPD Thruster [11]. This model is widely referred to as simply the Maecker formula:

$$T = \frac{\mu_0}{4\pi} \left[\ln \left(\frac{r_a}{r_c} \right) + \frac{3}{4} \right] J^2 \quad (7)$$

where r_a is the anode radius and r_c is the cathode radius [11]. Notably, when paired with a dimensionless thrust coefficient, the Maecker formula is independent of current density, mass flow rate, and propellant type. Regardless of its idealized nature, this equation has been applied many times to describe the scaling of the thrust of real

MPD thrusters [11]. However, as expected, the results shown by the formula did not agree with experimental thrust measurements [11].

An improvement to the Maecker equation is the Tikhonov equation, which is much closer to experimental values of thrust [11].

V. Experimental Performance and Design Features of Self-Field MPDTs

As mentioned before in this review, a self-field MPD Thruster consists of a central cathode within an anode. However, specific materials have not yet been discussed.

As per La Pointe and Gilland [12, 10], the anode is usually made of tungsten. Similarly, Gilland and Johnson [10] suggest tungsten infused with thorium or barium oxides to reduce the work for electron emission. The insulating backplate is made of boron nitride [10]. Often, hydrogen or inert gas is used for propellants. The theoretical ratio of the radius of the anode and the radius of the cathode is about 5.4, as suggested by the Princeton Benchmark Thrusters [10].

A variation in the design for an MPD Thruster is the nozzle. Nakata et al. [13] noted many different variations from around the world such as the straight, Stuttgart, Flared, Converging-Diverging, Centropazio, and the Princeton nozzles. However, the three main nozzle geometries considered were the straight, flared, and the convergent-divergent [13]. Within these geometries, they also considered degree of flare, cathode depth, and different throat configurations.

Through experiments, it was found that the convergent-divergent geometry was the most efficient nozzle. Specifically, the C-D geometry with a throat radius of 18mm and a throat length of 26mm [13]. With an argon mass flow rate of 0.80 g/s, currents between 2 and 15 kA, the optimal CD18-42 geometry delivered an efficiency of 30% [13]. Evidently, throat diameter served to be critical. A narrower CD configuration, CD12-42, produced reduced thrust [13].

V. PROPOSED LORTENZ RAIL ACCELERATOR DESIGN

I. Objective of The Design

Through the research conducted, a proposed Lorentz Rail Accelerator design was developed. With this design, the goal is to validate and analyze the behavior of the accelerator geometry using simulations. Specifically, the

analysis seeks to evaluate magnetic field vector distribution and flux density near the rails as functions of rail spacing. Thus, we can identify how this parameter influences the force in a Lorentz Rail Accelerator.

The current model considers the following: conductive rails, rail spacing, magnetic field, and flux density. That said, this model excludes the projectile motion aspect of the accelerator and mechanical stress or deformation. These effects are excluded to isolate electromagnetic design behavior and establish baseline estimates for magnetic field and flux density. Additionally, the software is limited in that it cannot evaluate the transient skin effects, which affects current crowding in the rail.

II. Simulation

The LRA model was modeled on the Fusion 360 CAD software. This model was analyzed on a finite-element solver capable of magnetoquasistatic simulations, namely FEMM. Therefore, I was able to analyze the distribution of magnetic flux and magnetic field vectors, which are related to the magnitude of the Lorentz Force. These results are valid for high-current, low-frequency applications, as seen in LRAs.

Using the FEMM software, I was able to import two-dimensional cross sections as DXF files from Fusion 360 to assign rail material, magnitude of current, while changing the rail spacing.

III. Results of Simulation

Upon the completion of iterative simulations, it was found that changing the rail spacing has considerable effect on the initial force vector in the rail apparatus. The initial spacing between the rails was set to be 10 mm, from which I reduced 1 mm per iteration. The result of the changing rail spacing on the peak magnetic flux density and average magnetic flux density is summarized in the table below.

TABLE I
EFFECT OF RAIL SPACING ON FLUX DENSITY

Rail Spacing vs Peak and Average Flux Density		
Rail Spacing (mm)	Peak B (T)	Average B (T)
5	1.28	1.18
6	1.23	1.12
7	1.18	1.06
8	1.16	1.02
9	1.14	0.99

10	1.13	0.93
----	------	------

Interestingly, the magnetic flux density is the highest near the rails, as shown in figure 1. Meaning, there are two spots in the gap between the rails that have peak magnetic flux density. This is contrasting with the assumption that was made earlier, which allowed for simple addition of the magnetic field vectors to create a larger vector acting from the middle of the rails. This idea is correct, but only in essence. Since there are two areas of peak magnetic flux density of equal magnitudes, the strength of the magnetic field changes as we get farther away from the rails.

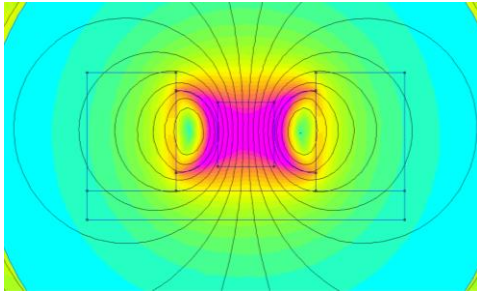


Fig. 1. Magnetic flux density (B) distribution across the rail-armature gap. Peak flux is concentrated at the rail interfaces

The initial Lorentz force on the armature was evaluated by numerically integrating the Lorentz force along the current path: $F = I \int_0^l d\vec{l} \times \vec{B}$. The magnetic flux density along the armature was obtained from the FEMM software analysis, and the resulting force for each iteration corresponds to the approximate force acting on the armature at $t \approx 0$ before motion and the diffusion of the current or the magnetic fields. To simplify calculations, $\vec{B}$ can be swapped with $\vec{B}_{avg}$ values that were found through the FEMM software. This makes the magnetic flux density term in the integral a constant of integration that can be pulled out.

TABLE 2
EFFECT OF RAIL SPACING ON LORENTZ FORCE AT $t \approx 0$

Rail Spacing vs Lorentz Force at $t \approx 0$		
Rail Spacing (mm)	Average B (T)	Lorentz Force (N)
5	1.18	118
6	1.12	134.4
7	1.06	148.4
8	1.02	163.2
9	0.99	178.2

10	0.93	186
----	------	-----

Interestingly, the force applied on the armature increases as the rail spacing increases. This is because the length of the conductor has a proportionally higher impact on the force than the localized reduction in magnetic flux density considering the 5mm and 10mm iterations, the length between the rails (and therefore the length of the armature) increased by 100%, as compared to the 21% decrease in the average magnetic flux density.

In addition, the influence of the inductance gradient has been established. Therefore, it is also worth examining the impact of rail spacing on the inductance gradient, and in turn, the total Lorentz force.

TABLE 3
EFFECT OF RAIL SPACING ON L' AND TOTAL LORENTZ FORCE

Rail Spacing vs L'		
Rail Spacing (mm)	Inductance Gradient (H/m)	Lorentz Force (N)
5	2.44×10^{-7}	48.8
6	2.58×10^{-7}	51.6
7	2.72×10^{-7}	54.4
8	2.84×10^{-7}	56.8
9	2.97×10^{-7}	59.4
10	3.07×10^{-7}	61.4

A comparative analysis between the values of the Lorentz Force in Table 2 and Table 3 shows a considerable decrease in force applied to the armature. However, there is a reason behind that discrepancy. The main distinction is that in Table 2, we are considering the force on the armature immediately after the current is applied, where not enough time has passed for phenomena such as magnetic field diffusion or armature motion. Table 2 shows the maximum theoretical impulse given on the armature at the start of the launch.

In contrast, the consideration of inductance gradient in Table 3 provides a deeper evaluation based on magnetic energy. Through the use of L' , Table 3 provides a more realistic average force for the whole duration of the acceleration. However, both tables demonstrated the same idea that an increase in rail spacing resulted in a net increase in the Lorentz Force acting on the armature, regardless of the drop in magnetic flux.

CONCLUSION

In conclusion, this work considered two applications of the Lorentz force: Lorentz Rail Accelerators and magnetoplasmadynamic thrusters. Although these systems operate in different applications and regimes, both derive their acceleration fundamentally from the interaction of moving currents or current densities with magnetic fields.

For LRAs, the analysis highlighted the dominant role of magnetic diffusion and eddy currents in inductance gradients and, consequently, the launch force. Though there may be an increase in magnetic flux density due to the changes of certain parameters, that does not directly translate to a higher force due to the various other factors that play a role in the rail accelerator. The analysis of transient skin effect clarified why rail geometry, rail spacing, and armature material directly govern energy efficiency and thermal loading. These factors impacted my design of a small-scale Lorentz Rail Accelerator.

For self-field MPD thrusters, the review focused on the unique application of the Lorentz force on plasma. Additionally, the review explored thrust models, their benefits, and their shortcomings. The survey of thruster geometry revealed insight on the difference nozzle geometry can have on the efficiency and thrust of the MPD Thruster, as Nakata et al. found with the specific configurations of the CD nozzle geometry that provided the most efficient results.

By comparing LRAs and MPDTS, this work provides a unified theoretical perspective by connecting solid-conductor and plasma-based acceleration. The results of this review highlight the importance that diffusion of physical properties and geometries form the critical set of parameters governing efficiency and scalability across both propulsion classes.

As interest grows in applications of electromagnetic propulsion (ground-based, and high-speed thrust for deep-space missions), leveraging understanding of the shared principles will be essential for developing next-generation acceleration apparatuses.

REFERENCES

- [1] M. Han and H. Rana, *Applied-Field Magnetoplasmadynamic Thrusters for Deep Space Exploration*. 2024.
- [2] Choueiri, Edgar. "Scaling of Thrust in Self-Field Magnetoplasmadynamic Thrusters." *Journal of Propulsion and Power*, vol. 14, no. 5, Sept. 1998, pp. 744–53.
- [3] Ehresmann, Manfred, et al. "Lunar Based Massdriver Applications." *ResearchGate*, Sept. 2016
- [4] Ray, Pradosh K. "ARC-DRIVEN RAILACCELERATORRESEARCH." NASA, 19890004074, Feb. 1987. NTRS
- [5] Ren, S.; Feng, G.; Zhang, P.; Li, T.; Zhao, X. Method of Calculating Inductance Gradient for Complex Electromagnetic Rail Launcher. *Electronics* 2022, 11, 2912
- [6] Ma, S.; Ma, H. Investigation on Dynamic Thermal Transfer Characteristics of Electromagnetic Rail Spray Cooling in Transient Processes. *Energies* 2025, 18, 5254.
- [7] Kim, B.-K., & Hsieh, K.-T. (1999). Effect of rail/armature geometry on current density distribution and inductance gradient. *IEEE Transactions on Magnetics*, 35(1), 413–416.
- [8] Kumar, S., & R. Murugan. (2018). Analysis of Inductance Gradient and Current Density Distribution Over Different Cross-section of Rails. *International Journal of Electrical and Computer Engineering (IJECE)*, 8(2), 723–729.
- [9] Gover, E. (2024). A Brief Overview on Magnetoplasmadynamic Thrusters. *ArXiv (Cornell University)*.
- [10] Gilland, J., & Johnston, G. (2007). MPD Thruster Performance Analytic Models. *NASA Technical Reports Server (NASA)*.
- [11] Choueiri, E. (1998). Scaling of Thrust in Self-Field Magnetoplasmadynamic Thrusters. *Journal of Propulsion and Power*, 14(5), 744–753.

[12] LaPointe, M. R., Strzempkowski, E., & Pencil, E. (2004). High Power MPD Thruster Performance Measurements. *40th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit*.

[13] Nakata, D., Toki, K., Funaki, I., & Arakawall, Y. (2005). Experimental verification of the nozzle shape optimization for Self-Field MPD thruster. *ResearchGate*.