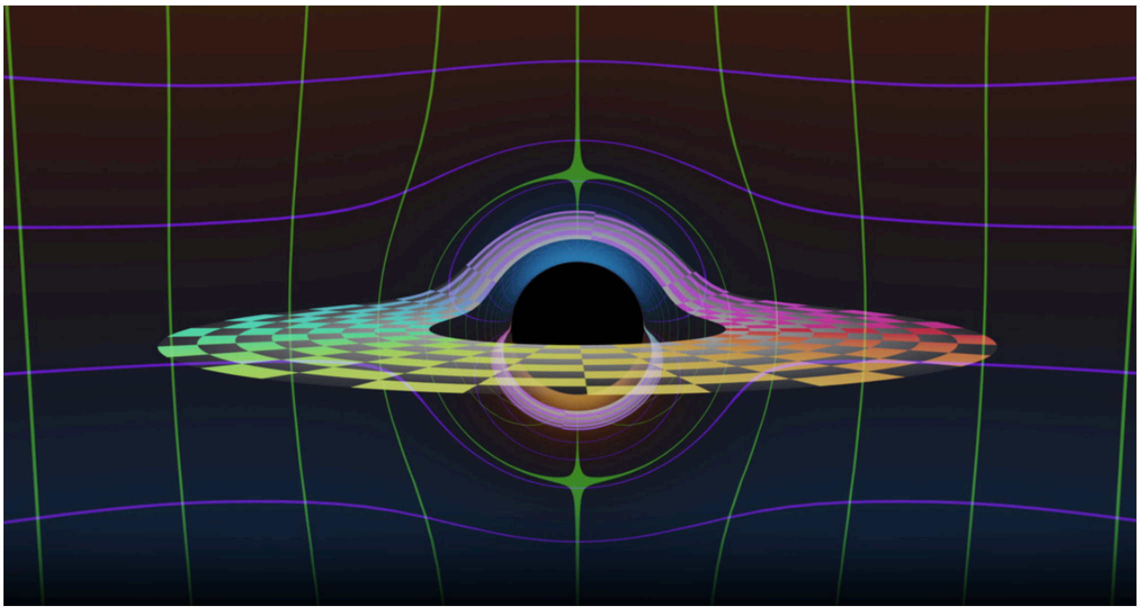
Computational Modeling of Kerr Black Hole Ringdown Oscillations

Adam Allam[1]*, Shaurya Singamsetty[1]*, and Shardul Singamsetty[1]*
Portola High, Irvine, CA, USA[1]
Contributed equally*



Kerr Black Hole visualization by Robert Hurt published on Nasa Scientific Visualization Studio[1]

Abstract

Our project investigates how the physical features of a Kerr black hole influence gravitational-wave ringdown oscillations after a binary black hole merger. Using a computational model based on quasi-normal mode approximations, the model generates inspiral, merger, and ringdown waveforms while allowing the final black hole's mass, dimensionless spin, mode composition, and detector noise level to be adjusted.[6,7] The simulation also used frequency-domain analysis, including a power spectrum, to identify dominant ringdown frequencies.[9] The results showed that increasing final black hole mass lowered the quasi-normal mode frequency and stretched the waveform over a longer timescale, while increasing spin produced more sustained oscillations with longer damping behavior times. The model also showed that detector noise distorted the waveform but did not fully remove the main ringdown structure. These findings support the hypothesis that Kerr black hole properties leave measurable signatures in gravitational-wave ringdown signals. Although the simulation is simplified and does not replace full numerical relativity or real detector analysis, it shows how computational modeling can connect general relativity, signal processing, and observable gravitational-wave behavior.[4,6] This project provides a visual way to explore how black hole mass and spin can be inferred from post-merger oscillations. [13]

Introduction

Gravitational waves are ripples in spacetime caused by the acceleration of large celestial bodies. Predicted by Albert Einstein's theory of general relativity, these waves carry energy away from collisions in space and travel outward at the speed of light.[2] Unlike electromagnetic radiation, gravitational waves give scientists a way to observe systems that may not emit visible light, making them especially important for studying dense objects such as neutron stars and black holes.[3]

One of the strongest sources of gravitational waves is a black hole merger. In these systems, two black holes orbit each other while gradually losing energy through gravitational radiation.[4] As energy is carried away, the black holes spiral inward, with an increasing orbital frequency. Subsequently, the emitted gravitational-wave signal forms a "chirp."[15] Eventually, the two black holes merge into a single, highly distorted final black hole.[4]

After the merger, the newly formed black hole undergoes a short period of damped oscillations known as the ringdown phase.[6] During this phase, the black hole stabilizes. These oscillations are described by quasi-normal modes, which behave like the fading vibrations of a bell.[7] Each mode has a characteristic frequency and damping time, meaning the ringdown signal contains information about features of the final black hole.[13]

Because most black holes are expected to rotate, the final system is modeled as a Kerr black hole.[5] Kerr black holes are described by their mass and angular momentum, often represented with a dimensionless spin parameter.[13] Changes in mass and spin affect the quasi-normal mode frequencies, damping times, and overall shape of the gravitational-wave ringdown.[6] For this reason, studying Kerr black hole oscillations is important for understanding how gravitational waves can reveal the features of black holes.[4]

This project tests whether a simplified computational model can produce the expected relationships between Kerr black hole properties and gravitational-wave ringdown signals. Using a numerical simulation based on quasi-normal mode approximations, we predicted that increasing final black hole mass would lower the dominant ringdown frequency and that increasing dimensionless spin would extend the damping time of the oscillations. We also predicted that detector noise would reduce waveform clarity while still allowing the dominant quasi-normal mode to be identified through frequency-domain analysis.[6,9] By comparing waveform shapes, frequency graphs, and mode tables across different parameters, our research shows how computational modeling can connect Kerr black hole physics to observable gravitational wave signals.

Key Terms

Gravitational Waves: Ripples in spacetime produced by the acceleration of extremely large objects, such as merging black holes or neutron stars.

Binary Black Hole Merger: An event where two black holes orbit each other, spiral inward, and combine into one larger black hole.

Ringdown: The final phase after a black hole merger, when the newly formed black hole's distorted spacetime settles down and produces fading gravitational waves which oscillate.

Quasi-Normal Modes (QNMs): The damped oscillation patterns, how a black hole vibrates during the ringdown phase. Each mode has its own frequency and damping time.

Kerr Black Hole: A rotating black hole described by the Kerr solution to Einstein's field equations. It is mainly characterized by mass and angular momentum.

Spin Parameter: A value that represents how quickly a black hole is rotating. In this project, changing spin affects ringdown frequency and damping behavior.

Damping Time: The time it takes for an oscillation's amplitude to decrease significantly. Longer damping time means the black hole "rings" for longer.

Frequency: The number of oscillations per second, measured in hertz. In terms of the ringdown phase, frequency is connected to the mass and spin of the final black hole.

Dominant Mode: The strongest quasi-normal mode in the ringdown signal. In this project, it refers to the main $l = m = 2$, $n = 0$ mode, which usually contributes the largest visible oscillation after merger.

Overtone: A higher-order quasi-normal mode that can appear early in the ringdown phase and usually decays faster than the dominant mode.

Time Domain: A way of viewing a signal by showing how it changes over time. In our project, the waveform graphs are time-domain because they show gravitational-waves as a function of time.

Frequency Domain: A way of viewing a signal by showing which frequencies are strongest in it. In our project, the power spectrum is a frequency-domain graph because it shows the main ringdown frequency peak.

Fast Fourier Transform (FFT): A signal-processing method used to change a waveform from the time domain to the frequency domain.

Spectrogram: A visualization showing how the frequency of a signal changes over time.

Detector Noise: Random background fluctuations in a detector that can make gravitational-wave signals harder to identify.

Signal-to-Noise Ratio (SNR): A measure of how strong a signal is compared to background noise. A higher SNR means the signal is easier to detect, and a lower SNR means the signal is more difficult to detect.

Perturbation: A small disturbance to a physical system. In our project, a black hole merger perturbs the final Kerr black hole, causing it to oscillate.

Numerical Simulation: A computer-based method for modeling physical systems using mathematical equations and algorithms.

Background Physics

Gravitational waves are one of the main predictions of Einstein's theory of general relativity.[17] In general relativity, gravity is not treated as a regular force that pushes or pulls objects together. Instead, large celestial structures such as black holes and planets curve spacetime, and other objects move through that curvature, creating orbits.[18] When extremely large objects accelerate, such as two black holes orbiting each other, they can create ripples in spacetime that radiate outwards at the speed of light.[2] These ripples are known as gravitational waves.

Black hole mergers are an especially strong source of gravitational waves because they involve extremely large masses moving at extremely high speeds in very compact regions of space.[4] A binary black hole merger is usually divided into three main stages: inspiral, merger, and ringdown.[10] During the inspiral stage, two black holes orbit each other while slowly losing energy through gravitational-wave radiation. As the orbit shrinks, the frequency and amplitude of the emitted waves increase, producing a signal known as a "chirp".[15] The merger stage occurs when the two black holes combine into a single distorted and unstable black hole.[4] The final stage is called ringdown. During this phase, the newly formed black hole is not yet stable. Instead, it vibrates as it settles into its final form. These vibrations are not random; they occur at specific frequencies known as quasi-normal modes.[7] A simple ringdown signal can be modeled as a damped oscillation, where the wave continues to oscillate while its amplitude decreases over time:[6]

$$h(t)=Ae^{-t/\tau}\cos(2\pi ft+\phi)$$

In this equation, $h(t)$ represents the gravitational-wave strain, A is the initial amplitude, f is the frequency of oscillations, τ is the damping time, and ϕ is the phase. The term $e^{-t/\tau}$ is exponential, which causes the signal to fade over time.[6] This behavior is similar and often described as the sound of a bell becoming quieter over time.

Quasi-normal modes are important because their frequencies and damping times depend on the properties of the final black hole.[7] In general relativity, an isolated black hole can be described mainly by its mass, electric charge, and angular momentum.[17] Since black holes are expected to have negligible electric charge, the most important features/parameters are mass and spin.[13] This means the ringdown signal can be used to estimate the physical properties of the final black hole.

Non-rotating black holes are described by the Schwarzschild solution, which lacks an electric charge and angular momentum described solely by their masses.[20] While rotating black holes are described by the Kerr solution, which lacks an electric charge but includes both angular momentum and mass.[5] Because black holes formed from mergers are expected to rotate, this project focuses on Kerr black holes.[4] A Kerr black hole's rotation is usually represented with a dimensionless spin parameter, often written as χ or a.[13] This parameter ranges anywhere from 0 as a static black hole to near 1 for an extremely rapidly rotating black hole.

The mass of a black hole strongly affects the ringdown frequency.[6] Larger black holes produce lower-frequency ringdown signals, while smaller black holes produce higher-frequency signals.[7] This inverse relationship can be summarized as:

$$f \propto \frac{1}{M}$$

where f is the ringdown frequency, and M is the black hole mass. Spin also affects the ringdown behavior. As the spin of a Kerr black hole changes, the quasi-normal mode frequencies and damping times change as well.[6] In most scenarios, higher spin allows the black hole to ring for longer before the signal fades.[13]

The damping time of a ringdown mode is often related to a quantity called the quality factor, written as Q.[6] The quality factor describes how many oscillations occur before the signal decays significantly. A higher Q value means the black hole rings for more cycles, while a lower Q value means the oscillation fades more quickly. The relationship between quality factor, frequency, and damping time is:[13]

$$Q=\pi f\tau$$

Ringdown analysis is important in gravitational waves because it provides a direct way to test whether the final object behaves like a Kerr black hole predicted by general relativity.[4] If the observed quasi-normal mode frequencies and damping times match Kerr predictions, the detection supports the theory.[6] If they're extremely different, it could suggest new physics or limitations in the model. For this reason, modeling Kerr black hole ringdowns is useful for understanding how gravitational-wave signals provide information about extreme spacetime systems.[7]

Methods

To investigate Kerr black hole ringdown behavior, we developed an interactive computational model that generates and analyzes gravitational-wave signals from simplified black hole mergers. The model is designed to represent the major phases of a binary black hole signal: inspiral, merger, and ringdown.[4] While the simulation is not a full relativity model, it uses approximations to show how black hole mass, spin, damping time, and quasi-normal mode structure affect the resulting waveform.[6]

Firstly, in the gravitational wave simulation, there is an inspiral phase where two black holes orbit each other and gradually move closer together as energy is emitted through gravitational waves.[4] In the simulation, this stage is represented by a chirp-like signal whose frequency and amplitude increase over time. This approximates the behavior seen in real gravitational-wave detection, where the orbital frequency rises rapidly before the black holes merge.[15] The point here is to provide the proper background for the ringdown stage and visualize the entire process of transformation of a binary into one black hole.

The merger phase represents a relatively short transition from the inspiral phase to the ringdown. During this phase, the two black holes combine into one distorted final black hole.[4] Because the actual merger requires complex relativity calculations, the model treats this stage as a smooth connection between the increasing inspiral signal and the decaying ringdown signal.[11] This allows the simulation to remain efficient while still showing the overall structure of a gravitational-wave event.

The ringdown phase is the key part of the model. It is generated using damped sinusoidal waves that represent quasi-normal modes of the final Kerr black hole.[6] Each mode is given a frequency, damping time, amplitude, and phase. The combined ringdown signal is calculated as a sum of these damped oscillations:[7]

$$h(t)=\sum_{n} A_n e^{-t/\tau_n}\cos(2\pi f_n t+\phi_n)$$

where A_n is the amplitude of each mode, f_n is the mode frequency, τ_n is the damping time, and ϕ_n is the phase. Including multiple modes shows that the ringdown signal can be made from several different fading oscillations, rather than one single wave.[7]

The model allows users to adjust physical parameters such as black hole mass and spin. The mass affects the overall frequency scale of the ringdown: larger black holes produce lower-frequency signals, while smaller black holes produce higher-frequency signals.[6] The spin parameter changes the quasi-normal mode behavior by affecting both the oscillation frequency and damping time.[13] This makes it possible to observe how rotating Kerr black holes produce different ringdown signatures from black holes that rotate less rapidly.

To analyze the simulated gravitational-wave signal, the model includes frequency-domain tools. A Fast Fourier Transform is used to convert the waveform from the time domain into the frequency domain, allowing dominant frequencies to be identified.[9] The simulator also includes a spectrogram, which shows how the frequency content of the signal changes over time.[16] This is especially useful for visualizing the inspiral chirp and the transition into the ringdown frequency after merger.

Detector noise is also included to make the simulation more realistic. Real gravitational-wave detectors such as LIGO must often identify very weak signals that are buried in background noise.[10] In the model, adjustable noise settings can be added to the waveform to show how detection becomes more difficult as the signal-to-noise ratio decreases.[16] This connects the theoretical model to the experimental challenge of measuring gravitational waves in real observatories.

Overall, the computational model combines simplified physics with signal processing.[6] By allowing black hole parameters to be changed and immediately visualized, the simulator provides a way to explore how Kerr black hole properties are encoded in gravitational-wave ringdown signals.[13] This makes the model useful both as a scientific visualization tool and as a method for analyzing the relationship between black hole physics and observable gravitational-wave behavior.

Results

The computational model generated simulated Kerr black hole gravitational-wave signals for different final masses, spin values, mode settings, and detector-noise levels. The baseline simulation used a final black hole mass of $M_f = 68.5\ M_\odot$, dimensionless spin $\chi = 0.69$, strain amplitude $h_0 = 10^{-21}$, and detector noise turned off.

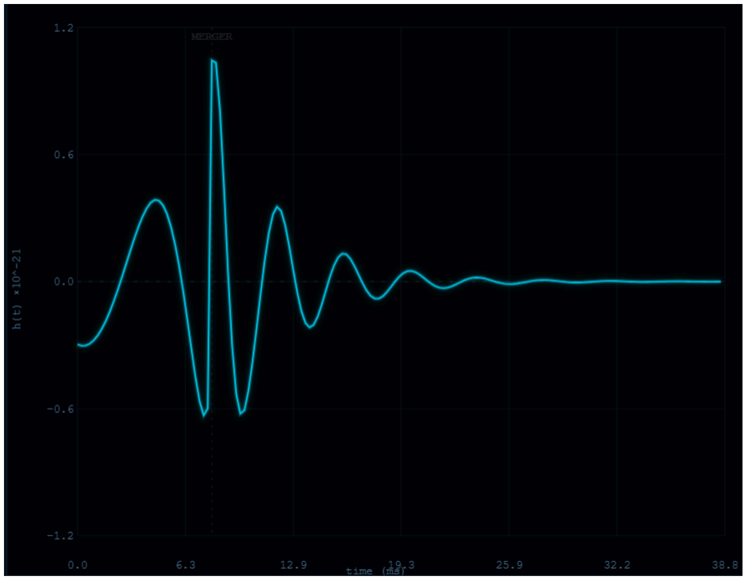


Figure 1. Simulated Kerr black hole gravitational-wave waveform showing inspiral, merger, and ringdown. Parameters: final mass M_f=68.5$M_\odot$, spin χ=0.69, inclination 23°, strain amplitude 10^{-21}, overtone setting n=0, detector noise off.

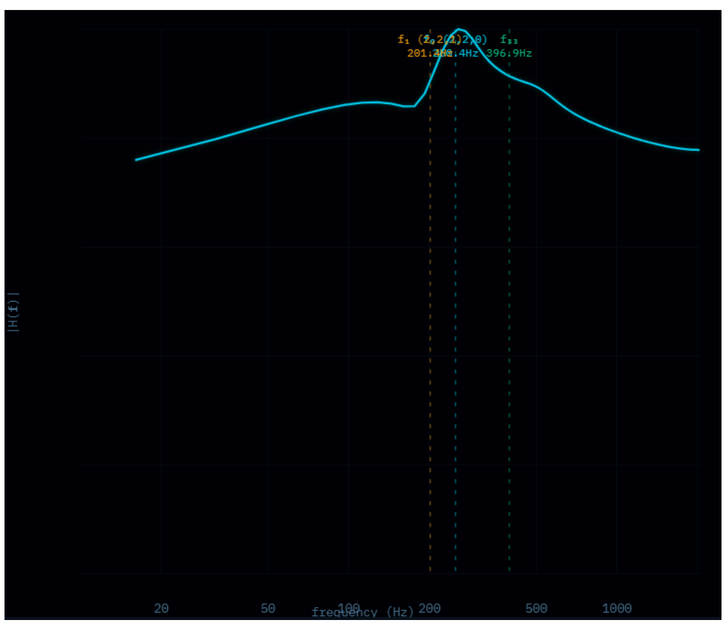
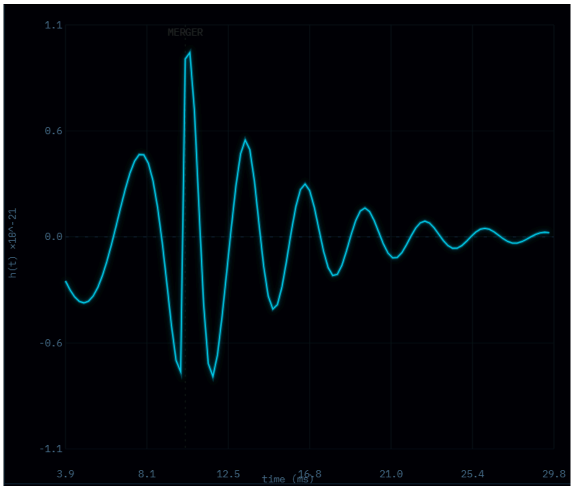


Figure 2. Power spectrum of the simulated Kerr black hole ringdown signal. The dominant peak corresponds to the main quasi-normal mode frequency, while the dashed vertical markers indicate predicted mode frequencies. Parameters: M_f=68.5$M_\odot$, χ=0.69, strain amplitude =10^{-21}, overtone setting n=0, detector noise off.



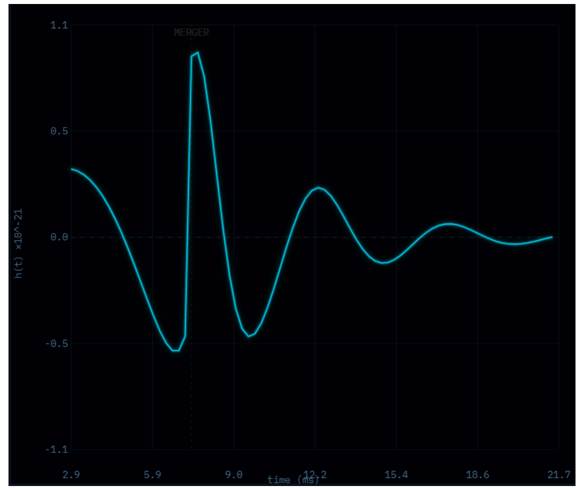


Figure 3a. High-spin Kerr ringdown waveform.

Figure 3b. Low-spin Kerr ringdown waveform.

Figure 3. Comparison of Kerr black hole ringdown waveforms at different spin values. With final mass held constant at M_f=68.5$M_\odot$, the higher-spin case shows more sustained oscillations, while the lower-spin case damps more quickly. Parameters: strain amplitude =10^{-21}, overtone setting n=0, detector noise off.

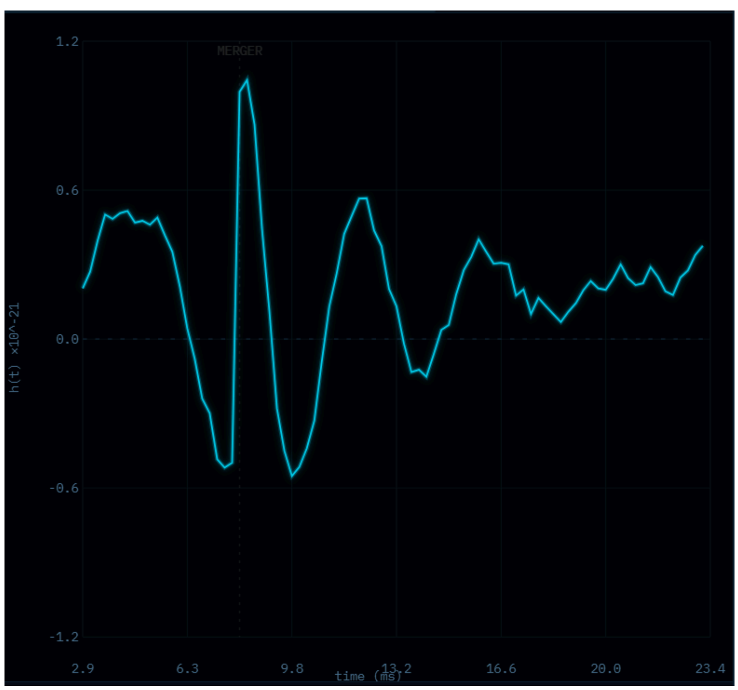


Figure 4. Simulated Kerr black hole ringdown waveform with detector noise added. The noise partially distorts the damped oscillations, demonstrating how detector noise can make gravitational-wave ringdown features harder to identify. Parameters: M_f=68.5$M_\odot$, χ=0.69, strain amplitude =10^{-21}, overtone setting n=0, detector noise on, noise level =5%.

Figure 5. Kerr parameter and mode table for the simulated black hole. The highlighted point shows the dominant quasi-normal mode frequency for M_f=68.5$M_\odot$ and χ=0.69, giving f_{QNM}=250.36 Hz and damping time τ=4.126 ms. The table also compares the dominant mode, first overtone, and l=m=3 subdominant mode, showing that different quasi-normal modes have different frequencies, damping times, and quality factors.

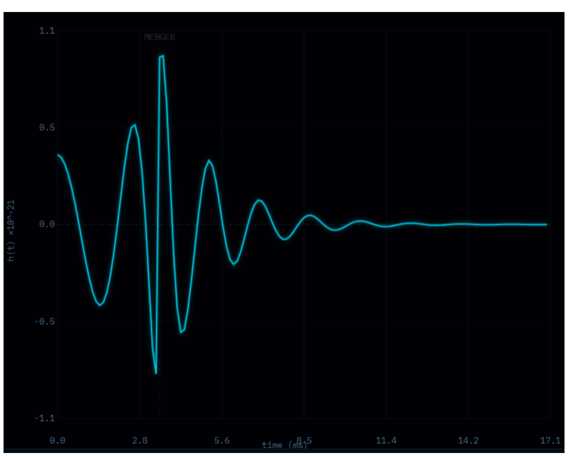


Figure 6a: low-mass case M_f=30$M_\odot$

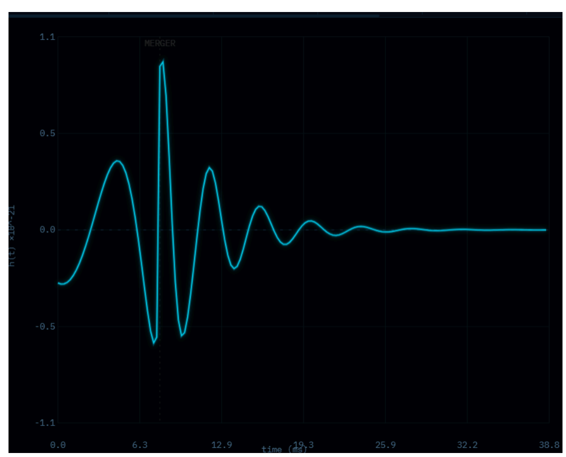


Figure 6b: intermediate-mass case M_f= 68.5$M_\odot$

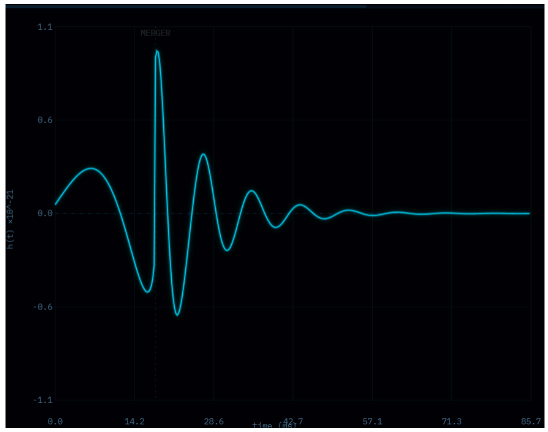


Figure 6c: high-mass case $M_f=150M_\odot$

Figure 6. Comparison of simulated Kerr black hole ringdown waveforms for different final black hole masses with spin held constant. As the final mass increases, the ringdown oscillations occur at a lower frequency and extend over a longer timescale, consistent with the inverse relationship between quasi-normal mode frequency and mass. Parameters: $\chi=0.69$, strain amplitude =10
−21, overtone setting n=0, detector noise off.

Analysis

The results support the hypothesis that Kerr black hole ringdown behavior depends strongly on the final black hole's mass and spin. Across the simulations, mass mainly controlled the frequency scale of the waveform, while spin mainly affected how long the oscillations remained visible after merger.[6] Detector noise did not remove the ringdown signal completely, but it made the waveform harder to interpret visually.[16]

The mass comparison in Figure 6 provides the clearest evidence for the inverse relationship between final black hole mass and ringdown frequency. When M_f increased, the oscillations became slower, and the waveform stretched across a longer interval. This corresponds with the expected relationship $f_{QNM} \propto 1/M$, because larger black holes have larger characteristic time scales.[6] In other words, a more massive final black hole "rings" at a lower frequency than a smaller one.

The spin comparison in Figure 3 shows that the dimensionless spin parameter χ changes the damping behavior of the ringdown. The higher-spin case produced more sustained oscillations, while the lower-spin case faded more quickly. This supports the idea that Kerr spin affects the quality factor and damping time of quasi-normal modes.[13] Because most black holes are expected to rotate, this result also shows why Kerr black holes are more realistic for modeling final black holes after the merger than non-rotating Schwarzschild black holes.[5]

The frequency-domain result in Figure 2 strengthens the model because it shows that the dominant quasi-normal mode is not only visible in the time-domain waveform, but also recoverable as a peak in the power spectrum. This is important because gravitational-wave analysis often depends on identifying frequency patterns inside noisy data.[16] The baseline value of f_{QNM} = 250.36 Hz demonstrates that the simulated waveform contains a measurable frequency connected to the final black hole's parameters.[6]

The mode table in Figure 5 shows that ringdown is not limited to a single oscillation. The dominant mode, first overtone, and $l = m = 3$ subdominant mode each have different frequencies, damping times, and quality factors.[7] This supports the idea that realistic ringdown signals are not just one simple wave, but a combination of several quasi-normal modes.[8]

The detector-noise test in Figure 4 connects the simulation to the experimental challenge of gravitational-wave astronomy. At a 5% noise level, the main ringdown pattern was still present, but the oscillations were less clear. This suggests that even when the signal does exist, detector noise can make it harder to identify.[10] Therefore, processing tools such as the Fast Fourier Transform and power spectrum are necessary for extracting ringdown information from noisy data.[9]

Overall, our model supports the main claim that Kerr black hole parameters leave measurable details encoded in gravitational-wave ringdown signals. Final mass affects the ringdown frequency, spin affects the damping behavior, additional modes change the signal structure, and detector noise reduces clarity. Although the simulation is simplified, its results match the expected trends of quasi-normal mode theory and show how computational modeling can connect black hole physics to observable gravitational-wave behavior.[6]

Conclusion

This project investigated how Kerr black hole properties affect gravitational-wave ringdown behavior through computational modeling. Using quasi-normal mode approximations, the simulation generated inspiral, merger, and ringdown waveforms for rotating black holes while allowing things such as final mass, spin, mode composition, and detector noise to be adjusted.[6]

The results show that black hole mass and spin have clear effects on the ringdown signal. Increasing the final mass lowered the ringdown frequency and stretched the waveform over a longer time interval, consistent with the relationship $f_{QNM} \propto 1/M$.[6] Changing the spin affected the damping behavior of the waveform, with higher spin producing longer oscillations.[13] The power spectrum confirmed that the simulated waveform contained identifiable quasi-normal mode frequencies, while the detector noise test showed how noise can make ringdown signals harder to recognize.[9]

Overall, our model demonstrated that gravitational-wave ringdowns carry important information about the final black hole after a merger.[4] Although our simulation is simplified and does not replace full relativity or real detector analysis, it successfully visualizes the major physical trends predicted by Kerr black hole perturbation theory.[11] This project shows how computational modeling can be used to connect advanced concepts from general relativity with visible gravitational-wave signals.

Improvements

Although our simulation captures the main behavior of Kerr black hole ringdown oscillations, it carries limitations that could be improved in future versions. The most important limitation is that the model is not a full relativity simulation. Real black hole mergers require solving Einstein's field equations, which is more complex than the simplified quasi-normal mode approximation used in this project.[11,12] Future work could improve the model by incorporating more advanced waveform data or comparing the simulation with relativity results.

The inspiral and merger phases are also simplified. In a real binary black hole merger, the waveform depends on the black holes' original masses, individual spins, orientation, distance from Earth, and viewing angle.[4] In this model, the inspiral is represented with a chirp-like approximation, and the merger is treated as a smooth transition into ringdown. A future version could include more realistic waveform models for each phase so that they match real gravitational-wave events.[15]

Another limitation is that the simulation includes only a small number of quasi-normal modes. The model focuses mainly on the dominant $l = m = 2$, $n = 0$ mode, with optional overtone and $l = m = 3$ subdominant mode behavior.[6] Real ringdown signals may contain additional modes depending on the merger conditions and observation angle.[7] Adding a larger set of modes would make the simulated ringdown more realistic and allow the model to represent a wider range of black hole merger scenarios.

The detector noise model is also simplified. Real gravitational-wave detectors experience many different types of noise, including seismic motion, environmental disturbances, thermal and quantum shot noise.[10] In this project, noise is used mainly to demonstrate how background fluctuations can distort the waveform. Future versions could use real detector sensitivity or open LIGO data to make the noise more accurate.[16]

Finally, the simulation does not perform full parameter estimation. In the current version, the user chooses black hole properties such as mass and spin, and the simulator generates the resulting waveform. A more advanced version could reverse this process by taking a waveform as input and estimating the final black hole's mass, spin, damping time, and dominant quasi-normal mode frequency.[13] This would make the model similar to real gravitational-wave analysis and would strengthen its usefulness as a research tool.

Evaluation

Our project successfully met its main goal of showing how Kerr black hole properties affect simulated gravitational-wave ringdown behavior. The model showed that final black hole mass changes the ringdown frequency, spin affects damping behavior, and detector noise makes the waveform harder to interpret. These results supported our hypothesis because the major trends in the simulation matched the expected behavior of quasi-normal modes.[6]

The strongest part of the project was the connection between physics and computation. Instead of only describing gravitational-wave ringdown theoretically, the simulation allowed us to change different parameters and compare them visually. The waveform plots, power spectrum, mode table, and noise test made the relationship between black hole properties and observable signals easier to analyze. This helped turn an abstract topic from general relativity into a model that could be tested and explained.

The sources used in this project were reliable overall. Peer-reviewed work by Berti, Cardoso, and Will, Abbott et al., Kerr, Leaver, and Teukolsky provided the scientific foundation for the model and background physics. Public sources from NASA, LIGO, and the Gravitational Wave Open Science Center were used mainly for background explanation, visualization, and real gravitational-wave context. Since the main claims were supported by scholarly sources, the research is stronger than if we had relied only on general web sources.

The main weakness of the project was that the simulation was simplified. It did not solve Einstein's field equations directly, use full relativity, and perform real parameter estimation from detector data.[11] Instead, it used quasi-normal mode approximations and simplified waveform generation.[6] This made the model easier to interpret, but it also limited how accurately it could represent an actual black hole merger.

Overall, the project was effective as a computational research model because it clearly showed the relationship between Kerr black hole parameters and ringdown signals. It also helped develop a better understanding of gravitational waves, quasi-normal modes, Fourier analysis, and the challenges of detecting weak signals in noisy data.[6] The project could be improved with more realistic waveform models, additional quasi-normal modes, and comparisons to real LIGO data.[16] However, the current version successfully demonstrates the core physics behind black hole ringdown oscillations.

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