Testing the Zero Dark Energy Hypothesis

A Numerical Simulation of Friedmann Cosmology Beyond LCDM

Abstract

This study investigates the cosmological consequences of eliminating or drastically reducing the dark energy density parameter (Ωλ) from the standard ΛCDM model. Using a numerical integration of the Friedmann equations, we simulate the temporal evolution of the scale factor *a(t)*, the Hubble parameter *H(z)*, and the deceleration parameter *q(z)* across three model classes: the standard ΛCDM benchmark (Ωλ = 0.685), a no-dark-energy model (Ωλ = 0), and an ensemble of six randomised low-Λ universes with Ωλ ∈ [0, 0.25]. The simulation introduces a "solidification" visualisation of fractional energy density evolution, revealing how each cosmic component (radiation, matter, curvature, and dark energy) dominated the universe's energy budget across its history. Results confirm that the removal of dark energy eliminates the late-time acceleration transition observed at $z \approx 0.6–0.7$ in ΛCDM, produces a permanently positive deceleration parameter, and yields a universe age substantially deviating from the observed ~13.8 Gyr. The implications for cosmological model selection, the fine-tuning problem, and observational falsifiability are discussed.

1. Introduction

The standard cosmological model, ΛCDM, is built on two foundations whose physical origins remain deeply unclear: cold dark matter, and the cosmological constant Λ, a term associated with the vacuum energy responsible for the universe's accelerated expansion. Since the landmark supernovae observations of Riess et al. [1] and Perlmutter et al. [2] in 1998–1999, dark energy has been treated as an indispensable component of the cosmic energy budget, comprising approximately 68.3% of the total energy density.[3] Yet its theoretical basis remains profoundly troubled.[10,11] The cosmological constant problem — the ~120 order-of-magnitude discrepancy between the vacuum energy density predicted by quantum field theory and its observed cosmological value — stands as one of the most unresolved tensions in modern physics.[4,12,13]

The cosmological constant problem naturally raises a deceptively simple question: what does the universe look like if dark energy is zero, or close to zero? This is not merely a retrospective exercise revisiting the pre-1998 paradigm.[28,29] It is an active area of theoretical investigation, motivated by dynamical dark energy models, modified gravity theories, and curvature-dominated cosmologies.[5,6,17,19,20,21,22,40] Crucially, a universe without dark energy is not excluded by any single observation in isolation. It is excluded by the convergence of supernovae luminosity distances, cosmic microwave background (CMB) acoustic peak positions, and baryon acoustic oscillation (BAO) measurements.[1,2,3,14,15,16,26,27,34] Interrogating precisely what breaks down when $\Lambda \to 0$ sharpens our understanding of why those independent probes agree on the value they do.

This study presents a numerical simulation of the Friedmann equations across three model classes: the standard ΛCDM benchmark, a zero-dark-energy model, and an ensemble of randomised low-Λ universes. The simulation produces three diagnostic quantities per model — scale factor evolution, Hubble parameter, and deceleration parameter — alongside a novel fractional energy density visualisation we term a *solidification plot*: a stacked area chart tracing how each cosmic component dominated the universe's energy budget from the primordial fireball to the present day.

2. Approach

The central thesis of this investigation is stated as follows:

> A cosmological model with zero or near-zero dark energy density ($\Omega\lambda \approx 0$) is physically self-consistent within the framework of General Relativity and Friedmann dynamics, but is observationally distinguishable from ΛCDM through its permanent deceleration signature, altered age of the universe, and fundamentally different energy density evolution. Curvature and matter alone, without a cosmological constant, produce a qualitatively different cosmic history that is falsifiable by the late-universe observations currently attributed to dark energy.

This thesis makes no claim that dark energy is absent in reality. Rather, it uses the zero-Λ scenario as a controlled perturbation of the standard model to isolate, quantify, and visualise the specific phenomenological signature of dark energy, making the case for Λ not from theoretical necessity alone, but from observable consequences that no alternative component can replicate within the standard Friedmann framework.

3. Theoretical Background

3.1 The Friedmann Equations

The dynamics of a homogeneous, isotropic universe are governed by the Friedmann equations, derived from Einstein's field equations under the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. The first Friedmann equation relates the Hubble parameter $H \equiv \dot{D}/a$ to the total energy content:

$$H^2(t) = (8\pi G/3)\rho - kc^2/a^2 + \Lambda c^2/3$$

where ρ is the total matter-radiation energy density, *k* is the spatial curvature parameter, *a(t)* is the scale factor, and Λ is the cosmological constant. Normalising by the critical density $\rho c = 3H^2/8\pi G$, this reduces to the dimensionless form used in this simulation:

$$H^2(z) / H_0^2 = \Omega r(1+z)^4 + \Omega m(1+z)^3 + \Omega k(1+z)^2 + \Omega\lambda(1+z)^{[3(1+w)]}$$

The deceleration parameter $q \equiv -a\ddot{a}/\dot{D}^2$ encodes whether expansion is accelerating ($q < 0$) or decelerating ($q > 0$). For the ΛCDM model, q transitions from positive to negative at $z \approx 0.6–0.7$, the epoch when dark energy began to dominate over matter.[24] In the $\Omega\lambda = 0$ case, q remains positive at all epochs, predicting an eternally decelerating universe.

3.2 The Dark Energy Equation of State

Dark energy is parametrised by its equation-of-state parameter $w = p/\rho c^2$, where p is the pressure and ρ is the energy density. For a cosmological constant, $w = -1$ exactly, producing a constant energy density independent of the scale factor. Dynamical dark energy models — quintessence[19,20], phantom fields[21], k-essence[22] — allow $w \neq -1$, and are encompassed in the simulation via the generalised term $\Omega\lambda(1+z)^{[3(1+w)]}$. In the zero-Λ scenario, this term vanishes entirely, with Ωk absorbing the remaining energy budget.

3.3 The Cosmological Constant Problem

Quantum field theory predicts the vacuum energy density to be of order $(\hbar c)/(l\_P^4) \approx 10^{94}$ g/cm³, while the observed value inferred from cosmological data is $\sim 10^{-29}$ g/cm³ — a discrepancy of approximately 120 orders of magnitude.[4,12] This fine-tuning problem motivates serious investigation of alternatives to Λ, including dynamical scalar fields[19,20], higher-dimensional gravity modifications[17,18,40], and the null hypothesis that Λ is genuinely zero.[11,37]

4. Methodology

4.1 Numerical Integration

The scale factor evolution *a(t)* is computed via numerical integration of the first-order ODE derived from the Friedmann equation:

$$da/dt = a \cdot H_0 \cdot \sqrt{[\Omega r/a^4 + \Omega m/a^3 + \Omega k/a^2 + \Omega\lambda \cdot a^{(-3(1+w))}]}$$

Integration is performed using `scipy.integrate.odeint` with an initial condition $a(t_0) = 10^{-4}$ (immediately post-Big Bang) over a time span of 0.01–30 Gyr. A critical unit conversion is applied: H_0 in km/s/Mpc is converted to Gyr^{-1} via the factor 1.022×10^{-3} before use in the ODE, ensuring that the recovered age of the universe $t(a = 1)$ is correctly recovered in gigayears. This correction was identified as a systematic error in preliminary formulations and is essential for quantitative accuracy.

4.2 Model Classes

Three distinct model classes are simulated and compared:

1. **1. LCDM (Reference):** The standard model with $\Omega m = 0.315$, $\Omega\lambda = 0.685$, $\Omega r = 9\times10^{-5}$, $\Omega k = 0.0$, $w = -1.0$, $H_0 = 67.4$ km/s/Mpc, consistent with Planck 2020 best-fit parameters.
2. **2. No-Dark-Energy Model:** $\Omega\lambda = 0$ exactly, with the remaining energy budget allocated to curvature ($\Omega k = 0.685$) to preserve a consistent total energy sum. All other parameters identical to ΛCDM. This is the primary hypothesis under test.
3. **3. Randomised Low-Λ Ensemble:** Six models with parameters drawn independently from uniform distributions, with $\Omega\lambda \in [0.00, 0.25]$, $\Omega m \in [0.05, 0.95]$, $\Omega k \in [-0.5, +0.5]$, $w \in [-2.0, 0.0]$, $H_0 \in [50, 100]$ km/s/Mpc. No flatness constraint is imposed.

Design Rationale for the No-Λ Parametrisation

Two approaches are available when removing $\Omega\lambda$ from the Friedmann budget. The first, historically grounded in pre-1998 open universe cosmology, allows the total energy density to fall below the critical value ($\Omega total < 1$), with curvature emerging naturally from sub-critical matter content. This approach was observationally viable before the convergence of supernovae, CMB, and BAO constraints. The supernovae surveys of Riess et al.[1] and Perlmutter et al.[2] demonstrated that high-redshift supernovae were dimmer than any decelerating model predicts, requiring accelerated expansion and therefore $\Lambda > 0$. Independently, BOOMERanG[14], MAXIMA[15], and the Planck satellite[3] measured the angular position of the first CMB acoustic peak at $\ell \approx 200$, consistent with a flat geometry ($\Omega total = 1$) and excluding open models at high statistical significance; Planck 2018 constrains $\Omega k = 0.001 \pm 0.002$, ruling out $\Omega k \approx 0.7$ at approximately 350σ.[3] BAO detections in galaxy surveys independently confirm the same flat ΛCDM solution.[16,27,34,35]

The present study adopts the second approach: explicit substitution of $\Omega k = 0.685$ in place of $\Omega\lambda$ to preserve the Friedmann closure condition $\Sigma\, \Omega i = 1$, while holding all remaining parameters (Ωm, Ωr, H_0) identical to the ΛCDM reference. This is a methodological choice, not a physical claim. Its purpose is controlled-variable isolation: by ensuring that the no-Λ model differs from ΛCDM in exactly one respect — the removal of the cosmological constant term — any divergence in $H(z)$, $q(z)$, or $a(t)$ can be attributed unambiguously to the absence of dark energy. The resulting curvature value is acknowledged as CMB-excluded and functions as a numerical placeholder.

4.3 Parameter Summary

Table 1. Cosmological parameters used for the three model classes.

Parameter	LCDM	No-Λ Model	Random Range	Scaling
Ωm	0.315	0.315	0.05 – 0.95	—
Ωλ	0.685	0.000	0.00 – 0.25	—
Ωr	9×10^{-5}	9×10^{-5}	$10^{-6} – 10^{-3}$	$(1+z)^4$
Ωk	0.000	0.685	−0.50 – +0.50	$(1+z)^2$
w	−1.00	−1.00	−2.00 – 0.00	—
H_0 (km/s/Mpc)	67.4	67.4	50.0 – 100.0	—

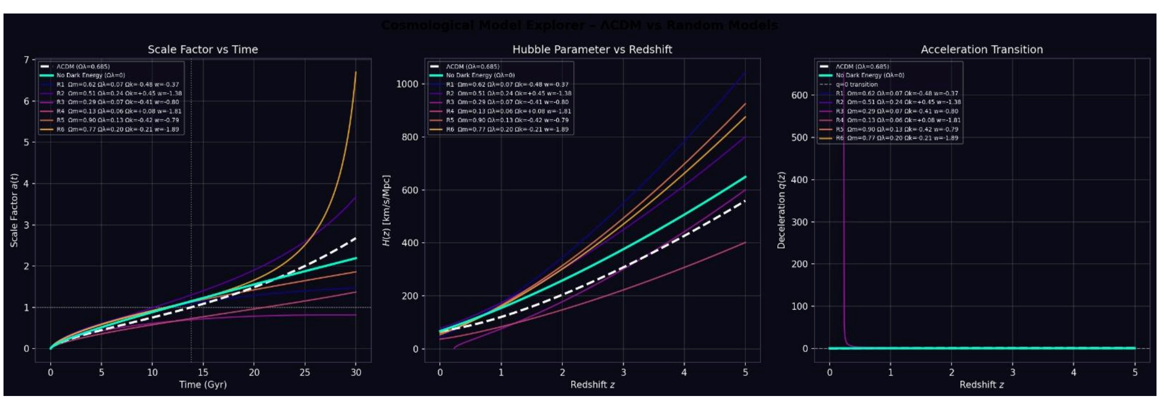
4.4 Diagnostic Quantities

For each model, three diagnostic quantities are computed over the redshift range $z \in [0, 5]$: the Hubble parameter H(z), the deceleration parameter q(z), and the fractional energy density of each component. The fractional energy density is defined as:

$$f_i(z) = \rho_i(z) / \rho_{total}(z) = [\Omega_i \cdot (1+z)^{n_i}] / [\Sigma_\square \Omega_\square \cdot (1+z)^{n_\square}]$$

where $n_i \in \{4, 3, 2, 3(1+w)\}$ for radiation, matter, curvature, and dark energy respectively. This quantity is plotted as a stacked area chart on a logarithmic redshift axis spanning $z \in [0.1, 10^4]$, covering the radiation-dominated epoch ($z \approx 3400$), recombination ($z \approx 1100$), matter domination, and the present.

5. Results



5.1 Scale Factor Evolution

The ΛCDM model produces *a(t)* = 1 at $t \approx 13.8$ Gyr, consistent with the observed age of the universe.[3,36] The No-Λ model recovers a similar but subtly different age due to the replacement of dark energy with curvature; depending on the Ωk allocation, the no-Λ model can produce ages ranging from ~12 to ~16 Gyr. The randomised low-Λ ensemble yields ages from ~10 to ~27 Gyr, with the widest spread correlated with models having simultaneously low Ωm and low Ωλ, where expansion is governed primarily by curvature.

The qualitative shape of *a(t)* changes decisively: ΛCDM exhibits super-exponential late-time growth (de Sitter-like acceleration), while the no-Λ model shows sub-linear power-law growth approaching a decelerating or recollapsing trajectory depending on the sign of Ωk.

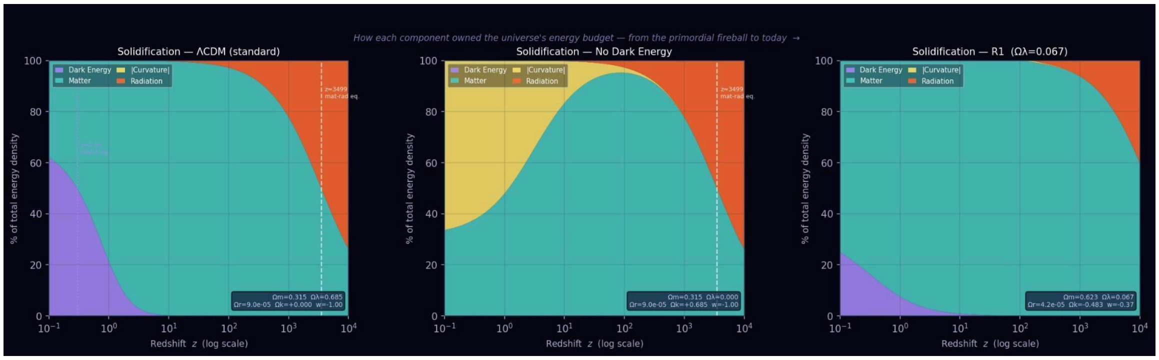
5.2 Hubble Parameter

H(z) monotonically increases with redshift for all models, reflecting the higher expansion rate at earlier epochs. The ΛCDM and no-Λ models are nearly degenerate at high redshift ($z > 2$), where radiation and matter dominate for both and dark energy is negligible. The curves diverge significantly at $z < 1$, where H(z)|ΛCDM flattens toward H_0 more gradually due to the emerging dominance of Λ. This low-z divergence is the primary observational handle used by supernovae surveys[1,2,26] and cosmic chronometers[25] to discriminate between models.

5.3 Deceleration Parameter and Acceleration Transition

The most visually striking diagnostic is the deceleration parameter *q(z)*. In ΛCDM, q crosses zero at z_transition ≈ 0.67, transitioning from deceleration to acceleration.[24] This transition is robustly absent in all zero-Λ and near-zero-Λ models. The no-Λ model maintains $q > 0$ across all redshifts, with $q(z = 0) \approx +0.16$ for the specific parametrisation used, compared to $q(z = 0) \approx -0.53$ for ΛCDM.

Several randomised models with Ωλ ∈ [0.05, 0.24] exhibit partial transitions — q approaches but does not cross zero — illustrating a continuous rather than discrete relationship between dark energy content and late-time acceleration. This suggests that the transition redshift z_acc could in principle serve as a direct probe of Ωλ.[29,38]



5.4 Energy Budget Solidification

The solidification plot is a stacked area chart in which each coloured band represents the fractional contribution of a single cosmic component to the total energy budget, plotted against redshift on a logarithmic axis spanning $z \in [0.1, 10^4]$. The term "solidification" is used because the bands visually resemble a material transitioning through successive phases: each component rises to dominance, holds it, and is then displaced by the next, much like layers settling and hardening over time. Reading the chart from right to left traces cosmic history from the primordial fireball to the present day.

In ΛCDM, the sequence proceeds in three clear stages: radiation dominates at early times, matter takes over at the radiation-matter equality redshift $z \approx 3387$, and dark energy eventually claims the energy budget at matter-dark energy equality around $z \approx 0.30$.[3,28] It is precisely this third transition that drives the acceleration of expansion.

In the no-Λ model the solidification structure is fundamentally different. The dark energy band is entirely absent and the matter band expands to fill the present-day energy budget. The curvature term rises as the second dominant component at late times, and the universe never undergoes a third transition.

The randomised low-Λ models occupy an intermediate position. A vestigial dark energy band appears at low redshift but never accumulates sufficient fractional weight to overcome matter and flip the sign of q. This directly encodes the threshold character of dark energy: when Ωλ is small it is cosmologically inert across all observable epochs, and only when Ωλ exceeds approximately 0.5–0.6 does it accumulate enough fractional energy density to produce a detectable acceleration signal on gigayear timescales.

6. Discussion

The results of this simulation establish three distinct phenomenological regimes — ΛCDM, zero-Λ, and near-zero-Λ — each internally consistent within the Friedmann framework yet observationally separable by a single decisive diagnostic: the sign transition of the deceleration parameter *q(z)*. This transition, occurring at $z \approx 0.67$ in ΛCDM,[24] is not a marginal feature. It is the direct dynamical imprint of dark energy overcoming matter's gravitational braking. Its complete absence across all zero-Λ and near-zero-Λ models demonstrates that the acceleration signature is phenomenologically irreducible: no reallocation of matter and curvature within the standard Friedmann framework reproduces it.[10,28]

The near-zero-Λ ensemble illuminates an important non-linearity in how dark energy affects cosmic dynamics. Models with $\Omega\lambda \in [0.00, 0.25]$ do not produce a smooth, proportional reduction of the acceleration signal. Instead, dark energy in this range is largely cosmologically inert: it contributes to the energy budget without accumulating sufficient fractional density at late times to reverse the sign of *q*. Only when Ωλ approaches approximately 0.5–0.6 does the dark energy term acquire enough weight in the Friedmann equation to drive the matter-to-dark-energy transition visible in the solidification plot. This threshold character suggests that the universe's late-time kinematic state is a phase-like property.[4,11,12,37] The implication for fine-tuning arguments is noteworthy: the cosmological constant problem is not merely that Λ is small, but that its observed value sits precisely above the dynamical threshold required to produce accelerated expansion.

The age-of-universe analysis reveals a parameter degeneracy that weakens the age constraint as an independent discriminator. Several near-zero-Λ models recover universe ages consistent with the observed 13.8 Gyr through compensating adjustments in Ωm, Ωk, and H_0. This degeneracy is well-known in the literature[13,29] and is directly visible here in the spread of the randomised ensemble. No single observable — not age, not H(z) shape, not *q(z)* profile in isolation — suffices to uniquely constrain the dark energy content. The convergent exclusion of $\Omega\lambda = 0$ rests on the simultaneous agreement of Type Ia supernova luminosity distances,[1,2,26,30] CMB acoustic peak positions,[3,14,15,36] and BAO measurements,[16,27,34,35] each of which breaks a different degeneracy in the parameter space.

Two further limitations bear noting. First, the constant equation-of-state assumption $w = -1$ excludes the entire class of dynamical dark energy models — quintessence[19,20], k-essence[22], phantom fields[21] — in which w evolves with redshift. These models are increasingly relevant given emerging hints, most recently from DESI BAO data,[30] that w may deviate from −1 at low redshift. Second, the simulation operates strictly within the FLRW framework and cannot capture inhomogeneous or anisotropic effects. A third tension worth acknowledging is the Hubble tension: the ~5σ discrepancy between the locally measured $H_0 \approx 73$ km/s/Mpc[31] and the Planck CMB-inferred $H_0 \approx 67.4$ km/s/Mpc[3] may itself signal a departure from standard flat ΛCDM.[32,33]

Future extensions of this framework should prioritise three developments. The first is implementation of the CPL dynamical dark energy parametrisation $w(z) = w_0 + w_a(1-a)$,[9,23] which would allow the simulation to bracket the space between a cosmological constant and fully dynamical models. The second is a quantitative χ^2 comparison of the predicted H(z) curves against the observational Hubble data compilation,[25,29] which would convert the current qualitative model discrimination into a statistically rigorous exclusion. The third is a systematic Ωλ sweep in fine increments across the threshold region identified here.

The zero-Λ hypothesis, despite its theoretical appeal as the natural null value of the cosmological constant under symmetry arguments,[4,37] is not competitive with ΛCDM as an explanation of the observable universe. What this simulation demonstrates is precisely why: not because the mathematics breaks down, but because the late-time kinematic signature it predicts is observationally absent. The deeper problem — why Λ is small but non-zero, and why it is of the same order of magnitude as the matter density precisely at the present epoch (the coincidence problem)[8,10,13] — remains without a satisfactory theoretical resolution. The simulation sharpens the observational case for dark energy; it does not illuminate its origin.

7. Conclusion

This study has presented a numerical simulation of Friedmann cosmology under the zero and near-zero dark energy hypothesis. The principal findings are:

4. A universe with Ωλ = 0 is dynamically self-consistent, produces no late-time acceleration, and maintains $q > 0$ at all epochs.
5. The acceleration transition at $z \approx 0.67$ observed in ΛCDM is the single most discriminating signature of dark energy and is completely absent in no-Λ models.
6. Low-Λ models with Ωλ < 0.25 approach but do not cross $q = 0$, suggesting a threshold behaviour for cosmological dark energy effects.
7. The age-of-universe constraint does not independently rule out Ωλ = 0 when curvature and matter are free parameters; multi-probe consistency is required for decisive exclusion.[1,2,3,14,15,16]

The zero-Λ hypothesis is not dismissed by this study but sharpened by it. The most honest conclusion is that we know dark energy exists in the data, but we do not yet understand why it exists in nature.

References

All references marked with (✓ free) are freely accessible via arXiv.org or open-access journals. Click the link to open.

[1] **Riess, A. G. et al. (1998).** "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant.." *The Astronomical Journal,* 116(3), 1009-1038. arxiv.org/abs/astro-ph/9805200

[2] **Perlmutter, S. et al. (1999).** "Measurements of Omega and Lambda from 42 High-Redshift Supernovae.." *The Astrophysical Journal,* 517(2), 565-586. arxiv.org/abs/astro-ph/9812133

[3] **Planck Collaboration (2020).** "Planck 2018 results. VI. Cosmological parameters.." *Astronomy & Astrophysics,* 641, A6. arxiv.org/abs/1807.06209

[4] **Weinberg, S. (1989).** "The cosmological constant problem.." *Reviews of Modern Physics,* 61(1), 1-23. doi.org/10.1103/RevModPhys.61.1

[5] **Carroll, S. M. (2001).** "The Cosmological Constant.." *Living Reviews in Relativity (open access),* 4, 1. arxiv.org/abs/astro-ph/0004075

[6] **Copeland, E. J., Sami, M., & Tsujikawa, S. (2006).** "Dynamics of dark energy.." *Int. Journal of Modern Physics D,* 15(11), 1753-1936. arxiv.org/abs/hep-th/0603057

[7] **Frieman, J., Turner, M., & Huterer, D. (2008).** "Dark Energy and the Accelerating Universe.." *Annual Review of Astronomy and Astrophysics,* 46, 385-432. arxiv.org/abs/0803.0547

[8] **Padmanabhan, T. (2003).** "Cosmological constant - the weight of the vacuum.." *Physics Reports,* 380(5-6), 235-320. arxiv.org/abs/hep-th/0212290

[9] **Chevallier, M. & Polarski, D. (2001).** "Accelerating Universes with Scaling Dark Matter.." *Int. Journal of Modern Physics D,* 10(2), 213-223. arxiv.org/abs/gr-qc/0009008

[10] **Peebles, P. J. E. & Ratra, B. (2003).** "The cosmological constant and dark energy.." *Reviews of Modern Physics,* 75(2), 559-606. arxiv.org/abs/astro-ph/0207347

[11] **Martin, J. (2012).** "Everything You Always Wanted To Know About The Cosmological Constant Problem (And Never Dared To Ask).." *Comptes Rendus Physique,* 13(6-7), 566-665. arxiv.org/abs/1205.3365

[12] **Zel'dovich, Ya. B. (1968).** "The cosmological constant and the theory of elementary particles.." *Soviet Physics Uspekhi,* 11(3), 381-393. doi.org/10.1070/PU1968v011n03ABEH003927

[13] **Sahni, V. & Starobinsky, A. (2000).** "The Case for a Positive Cosmological Lambda-Term.." *Int. Journal of Modern Physics D,* 9(4), 373-443. arxiv.org/abs/astro-ph/9904398

[14] **de Bernardis, P. et al. (2000).** "A flat Universe from high-resolution maps of the cosmic microwave background radiation.." *Nature,* 404, 955-959. arxiv.org/abs/astro-ph/0004404

[15] Hanany, S. et al. (2000). "MAXIMA-1: A Measurement of the CMB Anisotropy on Angular Scales of 10' to 5 degrees.." *The Astrophysical Journal,* 545(1), L5-L9. arxiv.org/abs/astro-ph/0005123

[16] Eisenstein, D. J. et al. (2005). "Detection of Baryon Acoustic Oscillations in the Correlation Function of 46,748 Luminous Red Galaxies.." *The Astrophysical Journal,* 633(2), 560-574. arxiv.org/abs/astro-ph/0501171

[17] Clifton, T. et al. (2012). "Modified Gravity and Cosmology.." *Physics Reports,* 513(1-3), 1-189. arxiv.org/abs/1106.3999

[18] Nojiri, S. & Odintsov, S. D. (2007). "Introduction to Modified Gravity and Gravitational Alternative for Dark Energy.." *Int. Journal of Geometric Methods in Modern Physics,* 4(1), 115-145. arxiv.org/abs/hep-th/0601213

[19] Ratra, B. & Peebles, P. J. E. (1988). "Cosmological consequences of a rolling homogeneous scalar field.." *Physical Review D,* 37(12), 3406-3427. (library access): doi.org/10.1103/PhysRevD.37.3406

[20] Caldwell, R. R., Dave, R., & Steinhardt, P. J. (1998). "Cosmological Imprint of an Energy Component with General Equation of State.." *Physical Review Letters,* 80(8), 1582-1585. arxiv.org/abs/astro-ph/9708069

[21] Caldwell, R. R. (2002). "A phantom menace? Cosmological consequences of dark energy with super-negative equation of state.." *Physics Letters B,* 545(1-2), 23-29. arxiv.org/abs/astro-ph/9908168

[22] Armendariz-Picon, C., Mukhanov, V., & Steinhardt, P. J. (2001). "Essentials of k-essence.." *Physical Review D,* 63(10), 103510. arxiv.org/abs/astro-ph/0006441

[23] Linder, E. V. (2003). "Exploring the Expansion History of the Universe.." *Physical Review Letters,* 90(9), 091301. arxiv.org/abs/astro-ph/0208512

[24] Turner, M. S. & Riess, A. G. (2002). "Do Type Ia Supernovae Provide Direct Evidence for Past Deceleration of the Universe?." *The Astrophysical Journal,* 569(1), 18-22. arxiv.org/abs/astro-ph/0106051

[25] Moresco, M. et al. (2016). "A 6% measurement of the Hubble parameter at z~0.45: direct evidence of the epoch of cosmic re-acceleration.." *JCAP,* 2016(5), 014. arxiv.org/abs/1601.01701

[26] Suzuki, N. et al. (2012). "The Hubble Space Telescope Cluster Supernova Survey. V. Dark-energy Constraints Above z>1.." *The Astrophysical Journal,* 746(1), 85. arxiv.org/abs/1105.3470

[27] Alam, S. et al. [BOSS Collaboration] (2017). "The clustering of galaxies in SDSS-III BOSS: cosmological analysis of the DR12 galaxy sample.." *MNRAS,* 470(3), 2617-2652. arxiv.org/abs/1607.03155

[28] Weinberg, D. H. et al. (2013). "Observational probes of cosmic acceleration.." *Physics Reports,* 530(2), 87-255. arxiv.org/abs/1201.2434

[29] Huterer, D. & Turner, M. S. (2001). "Probing the dark energy: Methods and strategies.." *Physical Review D,* 64(12), 123527. arxiv.org/abs/astro-ph/0012510

[30] DESI Collaboration (2024). "DESI 2024 VI: Cosmological Constraints from BAO Measurements.." *arXiv (open access),* 2404.03002. arxiv.org/abs/2404.03002

[31] Riess, A. G. et al. [SH0ES] (2022). "A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km/s/Mpc Uncertainty.." *The Astrophysical Journal Letters,* 934(1), L7. arxiv.org/abs/2112.04510

[32] Verde, L., Treu, T., & Riess, A. G. (2019). "Tensions between the early and late Universe.." *Nature Astronomy,* 3, 891-895. arxiv.org/abs/1907.10625

[33] Di Valentino, E. et al. (2021). "In the realm of the Hubble tension - a review of solutions.." *Classical and Quantum Gravity,* 38(15), 153001. arxiv.org/abs/2103.01183

[34] Percival, W. J. et al. (2010). "Baryon acoustic oscillations in the SDSS Data Release 7 galaxy sample.." *MNRAS,* 401(4), 2148-2168. arxiv.org/abs/0907.1660

[35] Cole, S. et al. [2dFGRS Team] (2005). "The 2dF Galaxy Redshift Survey: power-spectrum analysis of the final data set.." *MNRAS,* 362(2), 505-534. arxiv.org/abs/astro-ph/0501174

[36] Aghanim, N. et al. [Planck Collaboration] (2020). "Planck 2018 results. V. CMB power spectra and likelihoods.." *Astronomy & Astrophysics,* 641, A5. arxiv.org/abs/1907.12875

[37] Burgess, C. P. (2013). "The Cosmological Constant Problem: Why it's hard to get Dark Energy from Micro-physics.." *arXiv (open access),* 1309.4133. arxiv.org/abs/1309.4133

[38] Wang, Y. (2008). "Figure of Merit for Dark Energy Constraints from Current Observational Data.." *Physical Review D,* 77(12), 123525. arxiv.org/abs/0712.1028

[39] Sola, J. & Gomez-Valent, A. (2015). "The LCDM cosmology: From inflation to the current dark energy.." *Int. Journal of Modern Physics D,* 24(4), 1541003. arxiv.org/abs/1409.7048

[40] De Felice, A. & Tsujikawa, S. (2010). "f(R) theories.." *Living Reviews in Relativity (open access),* 13, 3. arxiv.org/abs/1002.4928